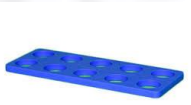
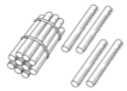
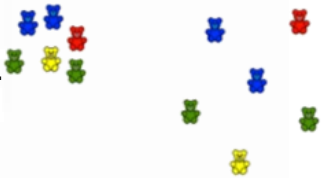
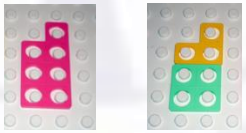
Introduction:

Children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **fluency** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use both manipulatives as well as pictorial representations (potentially as part of a **Concrete-Pictorial-Abstract – CPA – approach**) to support their mental and written methods of calculation. As children's mental methods are strengthened and refined, they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed.

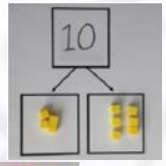
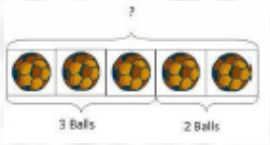
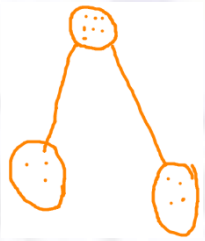
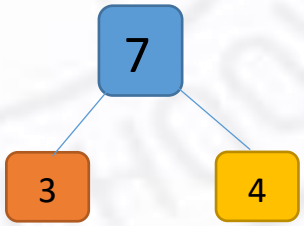
<u>From Early Years to Year 1:</u>

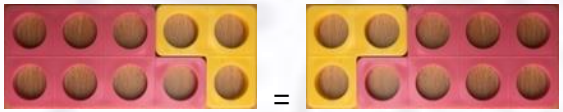
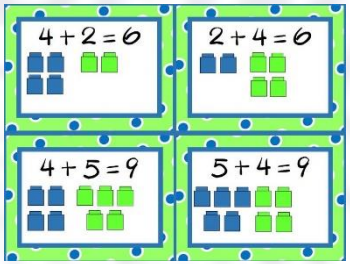
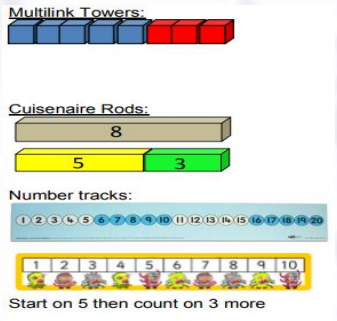
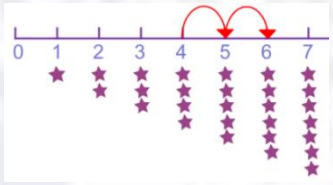
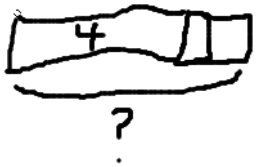
There are fundamental concepts that it is important for children to develop an early understanding of as building blocks to future learning in maths, including that linked to calculation. A selection of the skills include:

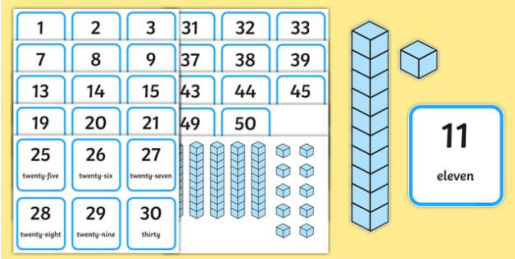
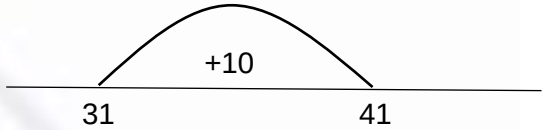
- Recognising and naming number symbols – (e.g. 5, 2, 4, 6…)
- Cardinality – 'understanding the value of different numbers' – e.g. (7 =  17 =  +  14 = 
- Ordinality – 'the ordering of numbers in relation to one another' – e.g. (1, 2, 3, 4, 5…) 
- Conservation of number – 'recognising that a value of objects are the same, even if they are laid out differently' – e.g. 
- One-to-one correspondence – e.g.  
- Concept of zero – e.g. 3 + 0 = 3
- Subitising – 'instantly recognising the number of objects in a small group, without counting them' – e.g.  → five
- Equality – 'seven is the same total as four add three' – e.g. 
- Counting on and back from any number – e.g. 'five add three more totals eight'  'ten take away three totals seven' 

Progression in Written Calculation

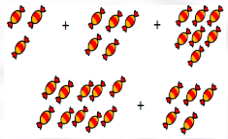
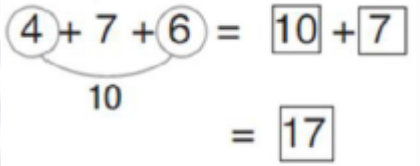
Addition. Addition names the whole in terms of parts, while subtraction names a missing part of the whole. Work must include a range of contexts, including those involving numbers, quantities and measures.

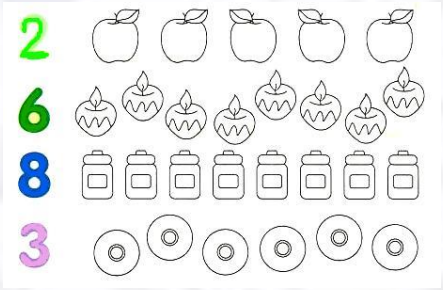
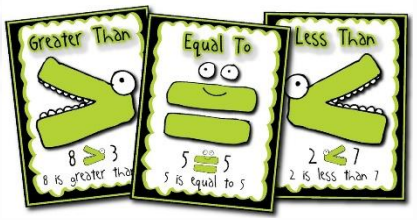
Objective and strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use objects to add two numbers together as a group or a bar (aggregation). Start counting at 1. Count one set and then the other. Then count them altogether. (Use other resources – eggs, teddies, shells, cars etc)  	 Use pictures to add two numbers together.  Use the **bar model** to add two numbers together. **whole** **part** | **part** Children draw crosses, dots or numbers in a part-whole model and add together.  	Children start to show recognisable abstract number sentences alongside the pictorial and concrete creations. Show that addition can be completed in any order. Introduce the **+** sign to replace 'and' in number sentences. Introduce the sign **= for *'is the same as'*** e.g. children begin moving from number sentences such as **4 and 3 are the same as 7** Shortening this to calculations that show the same **4+3=7**

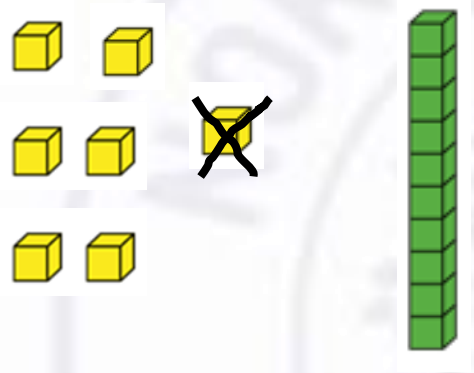
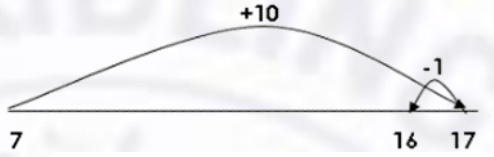
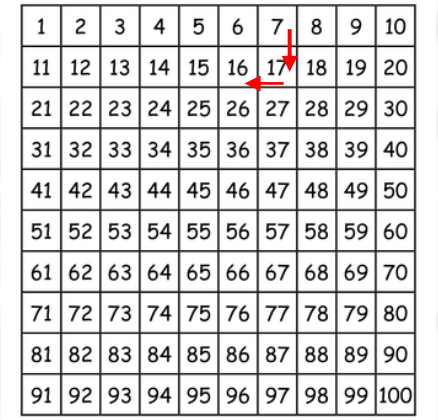
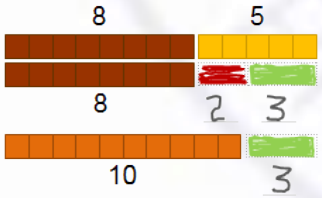
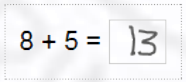
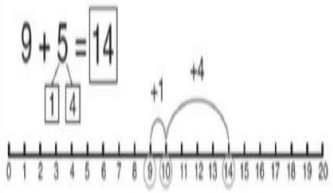
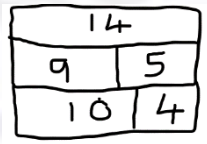
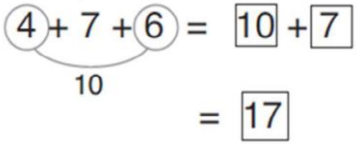
Objective and strategies	Concrete	Pictorial	Abstract
Adding in any order	Explore further the commutative law, where you can add in any order, using cubes or any real objects e.g make towers, comparing height to show that they are the same 	Draw the cubes to show that the total has not changed. 	Record calculations made with cubes. Progress to 3 digits and explore how starting at biggest is quicker 6 + 3 + 4 is the same as 6 + 4 + 3 however the second number sentence is easier & quicker. Children test this with other number sentences , looking for patterns and using bonds to find out which methods are quicker.
Starting at the bigger number and counting on	Children start to calculate rather than just count. Where one quantity is increased by some amount  (augmentation). Count on from the total of the first set (3 in your head) and count on 2. Encourage children to always start with the larger number. Use bead strings or Cuisenaire rods..	Use a number line with pictorial representation– start at the larger number and count on in ones.  Use a bar model that encourages the children to count on rather than count the whole.  Could also write number sentence along with bar or own creations.	Children continue to show recognisable abstract number sentences linked to number lines or tracks and are introduced to further addition vocabulary e.g. *'total', 'altogether' 'more than' 2 greater than 4 is'* etc **The sum of 4 and 2 is 6 2+4=6** **NB**: number lines are very different from number tracks which teach children the order of numbers. On a number line - points are marked instead (allowing fractions of numbers).

Objective and strategies	Concrete	Pictorial	Abstract
Adding 1s or 10s	Use cubes or dienes to start with a number and add on 10 more. 	Use or draw number squares to count on 1 more or 10 more by looking at the next number to the right or for 10 by looking directly below. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100	Children to use **empty number lines:**  Place larger number in your head and add on 10 more by counting in tens. Represent the number sentence in different ways: 41 = 31 +10, 31 + 10 = 41
Children should be here by the end of year 1			

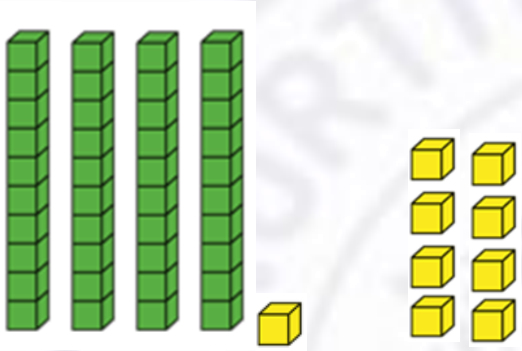
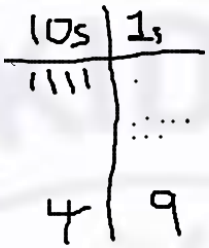
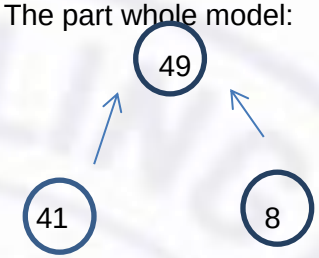
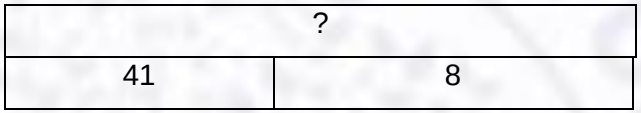
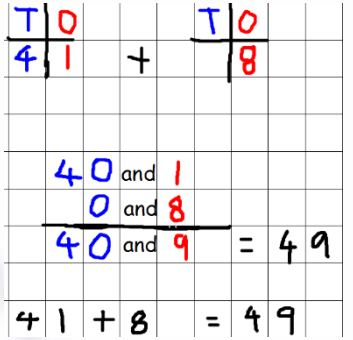
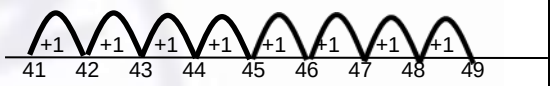
<table>
<tr><th colspan="4">Year 2</th></tr>
<tr><th>Objective and strategies</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Adding three single digits</td>
<td>Use Cuisenaire rods to work out sums e.g. 4 + 7 + 6 = 17. Put 4 and 6 together to make 10.
Add on 7.
 

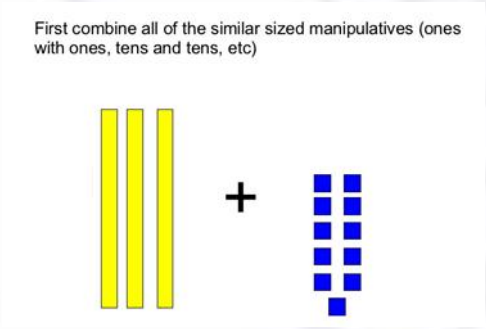
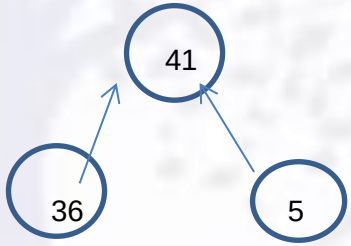
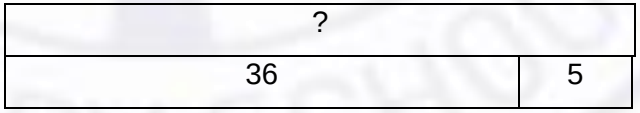
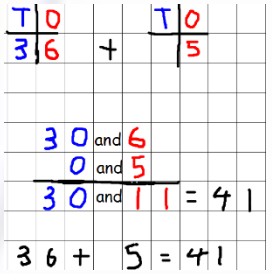
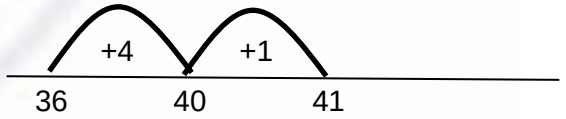
Build a tower of bricks and then ask the child to split them in 3 ways – add the numbers together.</td>
<td>Add together three groups of objects. Draw a picture to recombine the groups to make 10.
 
Bar models represent the 3 numbers.
<table>
<tr><td colspan="3">?</td></tr>
<tr><td>3</td><td>7</td><td>5</td></tr>
<tr><td colspan="2">10</td><td>5</td></tr>
</table></td>
<td>Combine the two numbers to make 10 and then add on the remainder.
 

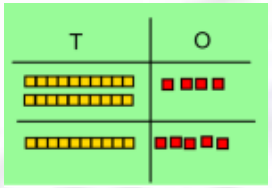
Children should be shown missing number sentences.</td>
</tr>
<tr>
<td>use the language of: equal to, more than, less than (fewer), most, least.</td>
<td>Use real objects to count and compare, model sentences for children to emulate.
 
</td>
<td>Children create their own pictures to show more/ less and verbally describe their pictures using correct terminology.
 
</td>
<td>Signs for equality and inequality
7=4+3 7=3+4
Children should begin comparing numbers using < and > signs.
 
</td>
</tr>
</table>

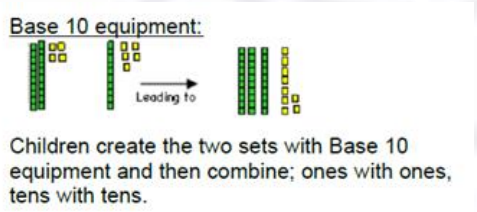
Objective and strategies	Concrete	Pictorial	Abstract
Adding 10 and then compensating	Use dienes to work out calculations e.g. 7 + 9 Children find 7 add 10 and then adjust by removing 1. 	Children draw a number line or use a 100 square to show the compensation.  	Writing number sentences to show process used, reinforced by other ongoing place value work. 7 + 10 = 17 17 – 1 = 16 7 + 9 = 16
Regrouping to make 10 then add on	Use Cuisenaire rods to show 8 + 5 can be partitioned into 8 + 2 + 3 (children use number bonds to 10).  	Children then draw to represent the regrouping – making a ten Use a number to partition (decompose) **e.g. 9 + 5**  OR… use their own pictures to show regrouping.  	Children record sentences made for regrouping on number lines in pictorial stage.  Combine the two numbers that make 10 and then add on the remainder.

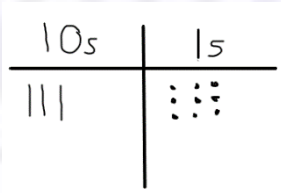
<table>
<tr>
<td>TO + O
(No regrouping)</td>
<td>Continue to develop understanding of place value and partitioning e.g. 41 + 8

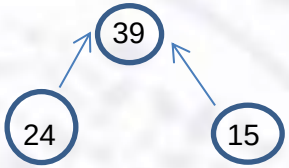
Using dienes or Cuisenaire rods to show bar models.
  </td>
<td>Represent base 10 with lines / dots
e.g. 41 + 8
 
The part whole model:
 
The bar model:
  </td>
<td>Children to use informal partitioning method:
 
or empty number lines
  </td>
</tr>
<tr>
<td>TO + O
(With regrouping in the ones)</td>
<td>Continue to develop understanding of partitioning and place value.

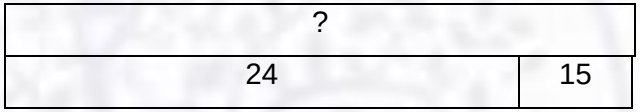
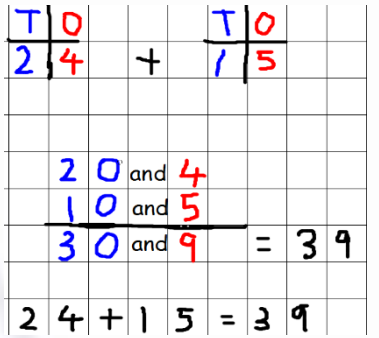
Using dienes or Cuisenaire rods to show bar models.
  </td>
<td>Represent base 10 with lines / dots
e.g. 36 + 5
The part whole model:
 
The bar model:
  </td>
<td>Children to use informal partitioning method:
 
or empty number lines
  </td>
</tr>
</table>

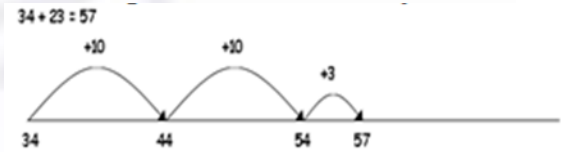
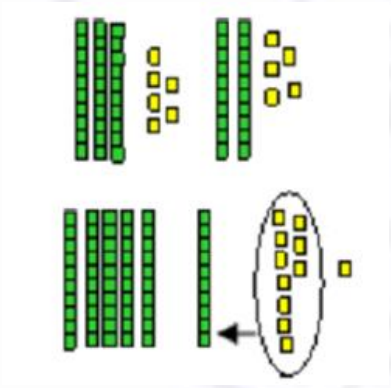
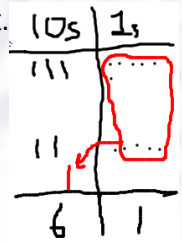
<table>
<tr>
<td>TO + TO
(No regrouping)</td>
<td>Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.
E.g. 24 + 15
 

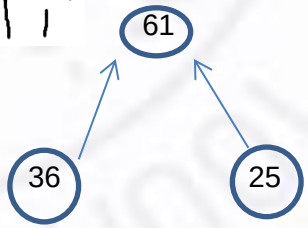


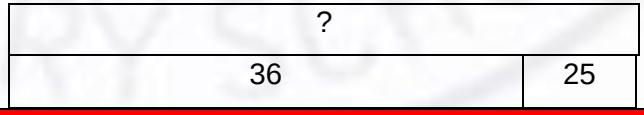
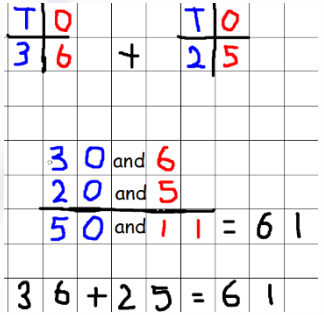
Using dienes/Cuisenaire to show bar models.</td>
<td>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.
 



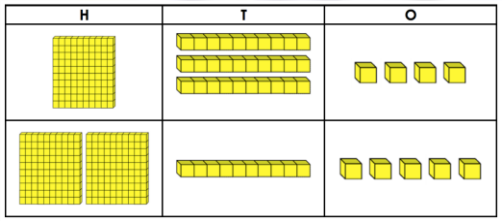
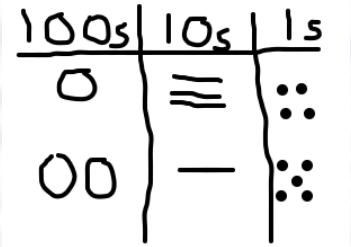
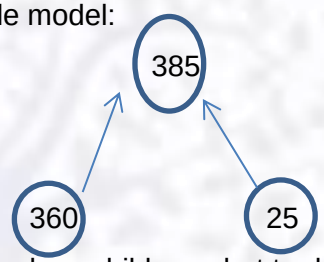
The bar model:
 
</td>
<td>Children to use informal partitioning method:
 

or empty number lines
 
</td>
</tr>
<tr>
<td>TO + TO
(With regrouping in the ones)</td>
<td>Show regrouping using dienes: 36 + 25
 
What are the missing counters and digits problems.
Use Cuisenaire rods to show bar models.</td>
<td>Children to represent base 10 by drawing it in a place value chart.
 

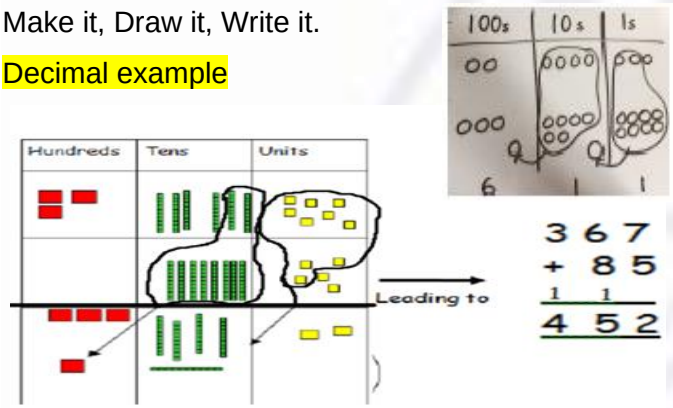
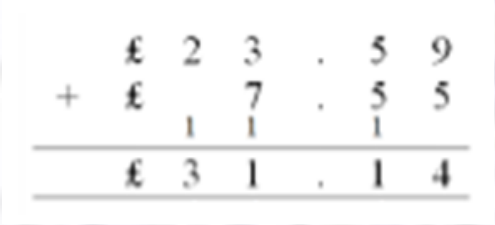
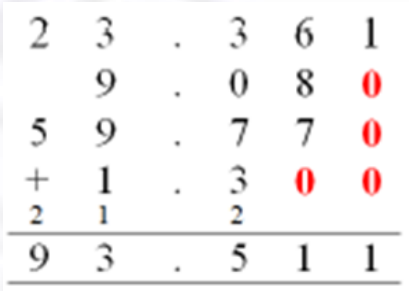
The part whole model:
 

The bar model:
 
</td>
<td>Children to use informal partitioning method:
 
</td>
</tr>
<tr>
<td colspan="4">Children should be here by the end of Y2</td>
</tr>
</table>

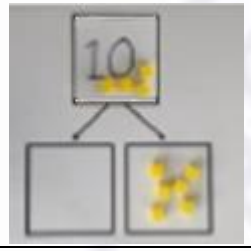
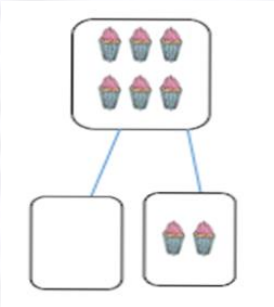
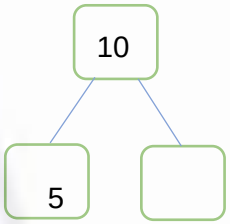
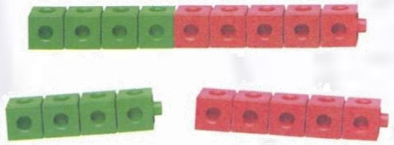
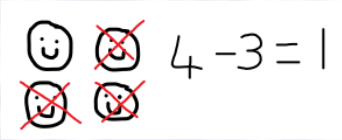
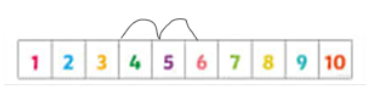
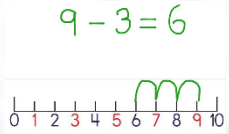
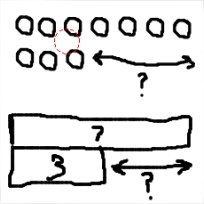
Year 3

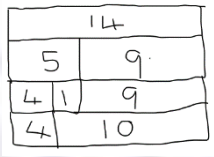
Objective and strategies	Concrete	Pictorial	Abstract
HTO + O (No regrouping) **HTO + O (With regrouping)** **HTO + TO (No regrouping)** **HTO + TO (With regrouping in the tens)** **HTO + TO (With regrouping in ones & tens)**	This can also be done with place value counters or Base 10. 134 + 215 =  Use Cuisenaire rods to show bar models. ? (sum) 134 | 215	Pictorial representation of the columns and counters. 134 + 215 =  The part whole model:  Bar models to show children what to do in a calculation. ? (sum) 360 | 25	Recap the **partitioning column method.** Introduce the **expanded column method.** 40 7 30 2 70+9= 79 Where the total in the 'Ones' is more than 9 children should be shown to move the 10s into the relevant column e.g. **47+39= 86** 40+7 30+9 **10** 80+6=86 **Children should learn to leave a space below the calculation ready for any 'carrying over'**

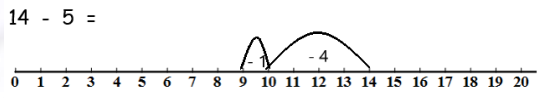
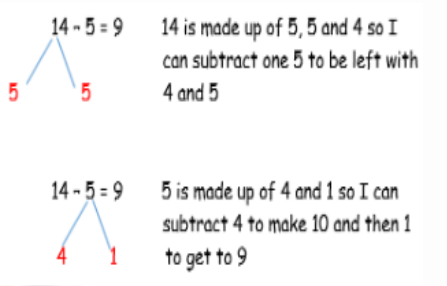
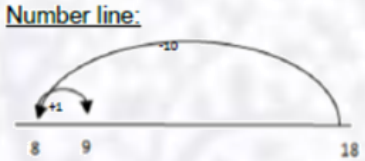
Children should be here by the end of Y3

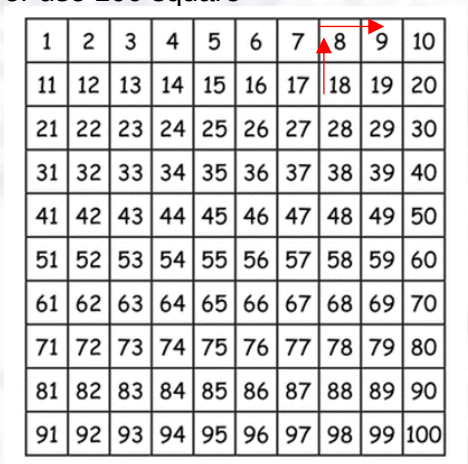
Year 4-6		
Y4 – **compact column method** up to 4 digits and two decimals (introduced with money) with the same number of digits. If there are more than 10 or more counters in a column regroup into a new counter in the next column. Then add up all the columns. Make it, Draw it, Write it. Decimal example 	**Y5** – **compact column method** with more than 4 digits and decimals with different number of digits and regrouping in some columns. Make it, Draw it, Write it.  Use place value counters for adding decimals	**Y6** – **compact column method** with more than four digits and decimals with different place value and regrouping in some columns. Make it, Draw it, Write it.  Use place value counters for adding decimals

Subtraction

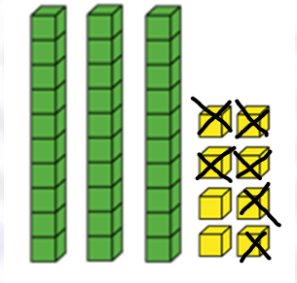
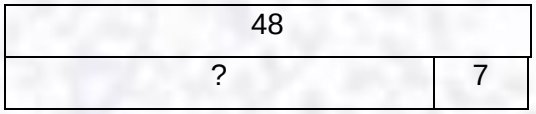
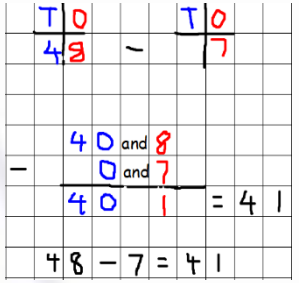
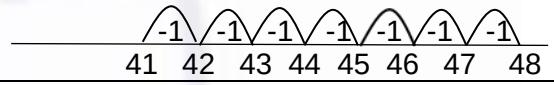
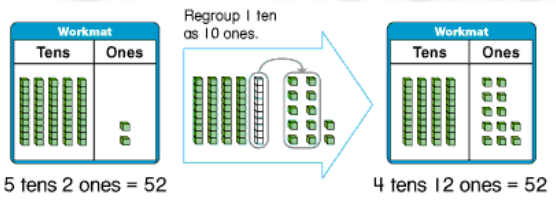
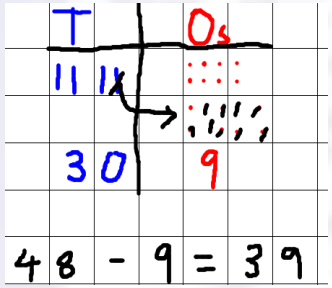
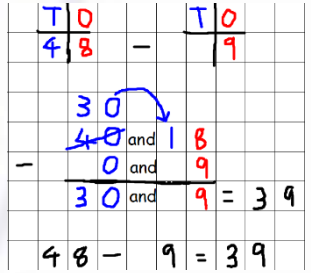
Objective and strategies	Concrete	Pictorial	Abstract
Part whole model	Link to addition – use the part whole model to help explain the inverse. Explore using counters and counting objects. 	Use a pictorial representation of objects 	Use numbers within the part whole model 
Taking away ones	Use physical objects: counters, cubes etc to show how objects can be taken away (separation model). 	Cross out drawn objects to show what has been taken away 	Children start to show recognisable abstract number sentences. 4 – 3 = 1
Counting back	Using number lines or number tracks. Children start with 6 and count back 	Represent on number line (full and empty). 	Children start to show recognisable abstract number sentences. 6 – 2 = 4
Find the difference	Finding the difference using cubes, Numicon or Cuisenaire rods (comparison model). 	Draw the cubes. Use the bar model. 	Find the difference between 8 and 5 8 – 5 = Explore why 9 – 6 = 8 - 5

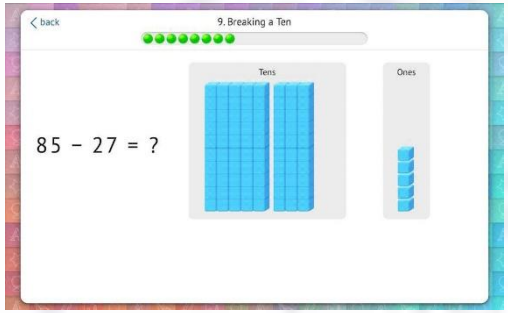
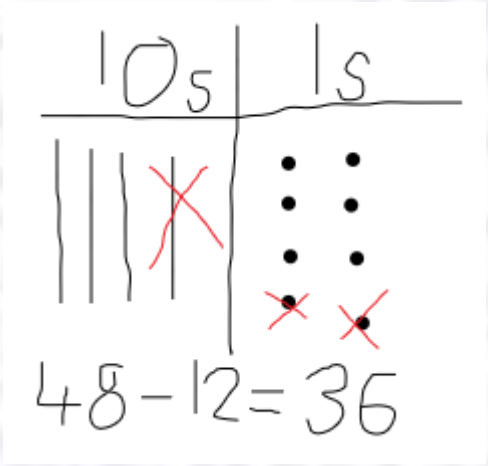
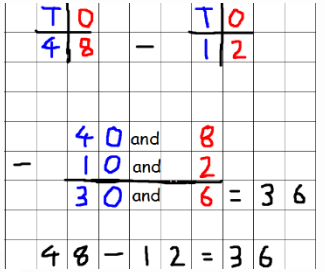
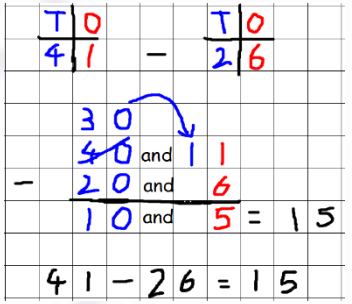
<table>
<tr>
<td>Make 10 (bridging 10) by partitioning one of the numbers</td>
<td>14-5 (Numicon, counters, 10 square)
 
Take away 4 to make 10
Then takeaway 1 so you have taken away 5.
You are left with the answer of 9.</td>
<td>Bar model:
 

Number line. Start at 14. Partition the next number. Take away 4 to reach 10. Take away 1.
 
</td>
<td>Children start to show recognisable abstract number sentences. 14 – 5 = 9
 
</td>
</tr>
<tr>
<td>Subtracting 10 and then compensating</td>
<td>18 – 9
 
Children find 18 then subtract 10 and then adjust by adding 1.
  </td>
<td>Children draw a picture to show the compensation
 

or use 100 square
 
</td>
<td>Writing number sentences to show process used, reinforced by other ongoing place value work.
18 - 10 = 8
8 + 1 = 9
18 - 9 = 9</td>
</tr>
<tr>
<td colspan="4">Children should be here by the end of Year 1</td>
</tr>
</table>

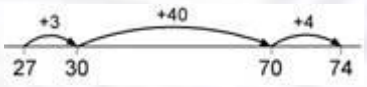
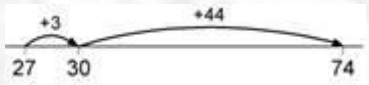
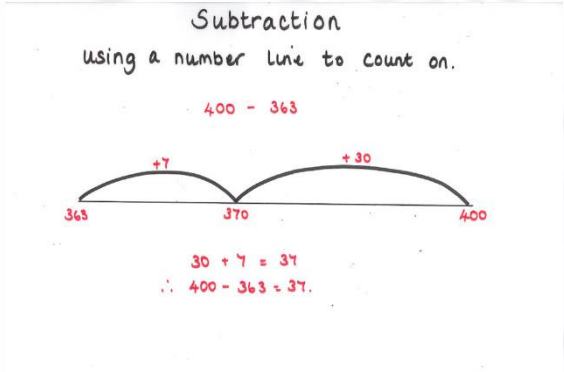
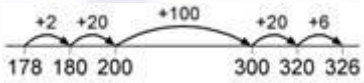
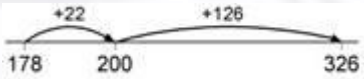
Year 2

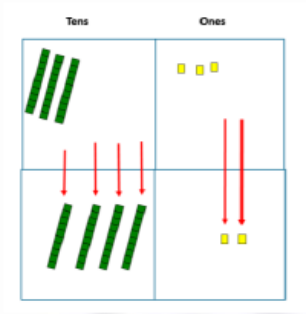
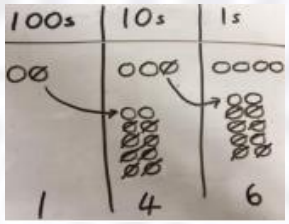
Objective and strategies	Concrete	Pictorial	Abstract
TO – O **(No regrouping)**	Create the bigger number using dienes and then subtract the smaller number. 38 – 6 = 32 	Draw the dienes and then cross out what you are subtracting.  The bar model:  	Introduction of the **partitioning column method:**  Or use **empty number line** 
TO – O **(With regrouping in the ones)**	Create the bigger number using base 10/place value counters and then subtract the smaller number. You can't remove 9 from 8, so you need to 'steal' a ten from the next column. Regroup into 30 and 18. Children can play around with numbers that can add to 48.  	Draw the base 10/place value counters and then cross out what you are subtracting. The regrouping must be clearly shown.  	Introduction of the **partitioning column method:** 

TO – TO	Create the bigger number using base 10/place value counters and then subtract the smaller number. 	Draw the base 10/place value counters and then cross out what you are subtracting. 	Introduction of the **partitioning column method – no regrouping:**  Introduction of the **partitioning column method with regrouping** 

Children should be here by the end of Year 2

Year 3

Objective and strategies	Concrete	Pictorial	Abstract
TO-TO subtraction by finding the difference between numbers and inverse operations to check answers.	Use table top marked number lines and plastic or laminated frogs. Show how to subtract by 'counting on' / finding the difference betweenfrom smaller to larger  74 27 | ? 27 + ? = 74 ? + 27 = 74 74 - ? = 27 74 – 27 = ?	Children record on their own blank number lines e.g. **74 – 27 =47**  **3+40+4= 47** or:  **3+44=47** The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shop keeper's method).	**Expanded subtraction -no decomposition needed:** **70** **Decomposition needed:** **60 10** **~~70~~ 4 -** **20 7**
HTO-HTO subtraction by finding the difference between numbers and inverse operations to check answers.	**using a number line**  	326 – 178 =  or:  	**Expanded subtraction** 300 20 6 100 10 4 200 10 2

HTO - O (No regrouping) **HTO - O (With regrouping)** **HTO - TO (No regrouping)** **HTO - TO (With regrouping in the tens)** **HTO - TO (With regrouping in ones & tens)**	 Use Base 10 to make the bigger number then take the smaller number away. **Move towards regrouping/ decomposition when bottom digit is higher than the one above.** Show how you partition numbers to subtract. Again make the larger number first.	Represent the place value counters pictorially; remembering to show what has been regrouped. 234-88  The bar model: 234 ? | 88	**Expanded subtraction-no decomposition** **e.g.563-232** 500 60 3 200 30 2 200 30 1 = 231 **Decomposition :563-232** 500 50 ~~60~~ 10 3 200 30 4 300 20 9 =329
Children should be here by the end of Year 3			

Year 4-6

Y4 – compact column method with regrouping with up to 4 digits. Make it, Draw it, Write it.

Decimal example

	$^{1}\cancel{2}$	$^{12}\cancel{3}$	4
-		8	8
	1	4	6

1

Y5 – compact column method with regrouping with more than 4 digits. Make it, Draw it, Write it.

	2	$^{5}\cancel{6}$	$^{12}\cancel{3}$	.	$^{1}0$
-		2	6	.	5
	2	3	6	.	5

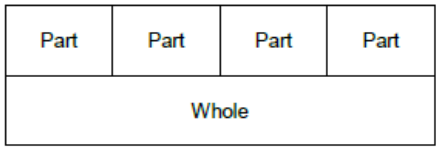
Y6 - compact column method with regrouping with more than 4 digits.
Place value counters for decimals with different amount of decimal places.
Make it, Draw it, Writ

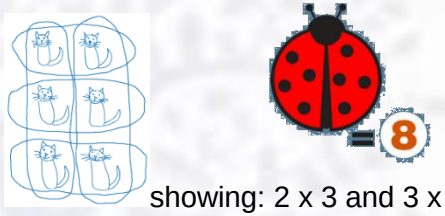
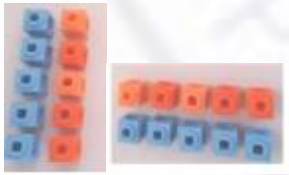
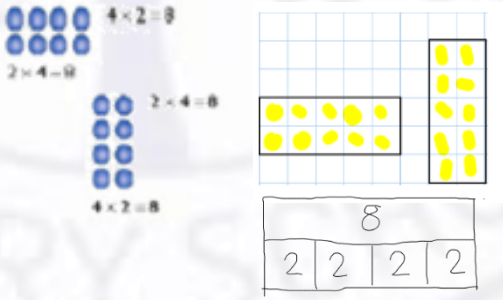
$$\begin{array}{r} {\scriptstyle 8\ \ 12\ \ 1} \\ 9\ \cancel{3}\ \cancel{2} \\ -\ 4\ 5\ 7 \\ \hline 4\ 7\ 5 \end{array}$$

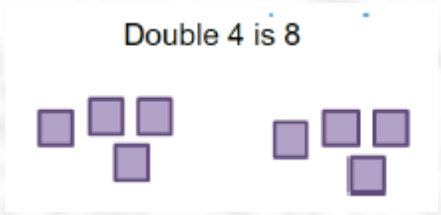
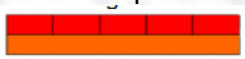
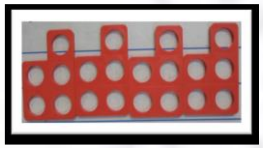
Answer: 475

Multiplication

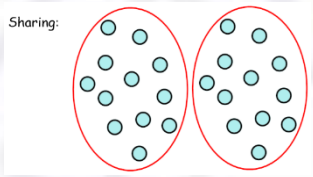
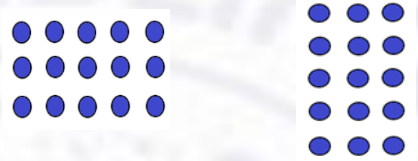
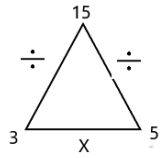
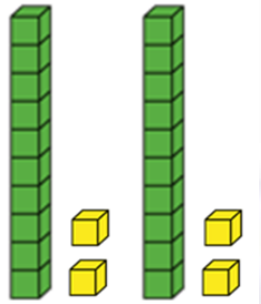
Multiplication and division are connected. Both express the relationship between a number of equal parts and the whole.



Objectives and strategies	Concrete	Pictorial	Abstract
Multiply by adding equal groups together	Use a set of objects. Double the set by finding the same number again. Make sure both sets are equal. 	Draw the objects   showing: 2 x 3 and 3 x 2	Children may start to show recognisable abstract number sentences. 3 x 4 = 12 4 + 4 + 4 = 12
Introduction of using arrays to count in multiples of 2, 5, 10 (commutative law)	Use a set of objects. Children can place them in groups or start to focus them in on array shapes. 2 x 5 5 x 2 	Draw the objects in arrays. Draw in different rotations to find the commutative sentences. This prepares children for the grid method and finding of factors. Also, to help find the area of rectangles. 	Children count in multiples of a number out loud. Write sequences with multiples of numbers. 2, 4, 6, 8 etc Children start to use an array to write a range of abstract calculations. 2 x 5 = 10, 5 x 2 = 10, 5 + 5 = 10, 2 + 2 + 2 + 2 + 2 = 10

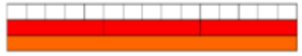
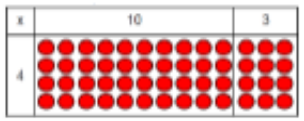
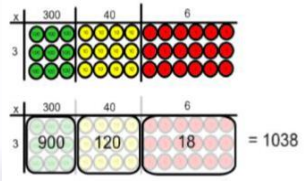
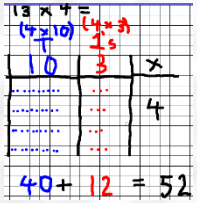
Reason about odd and even numbers and relate to doubling and halving	Create arrays of odd and even numbers with objects – what is the same or different about them? Double the number by adding the same number of objects and discuss what happens.	Draw the objects and circle/highlight the differences and similarities. Draw what happens when you double the number.	Children may start to show abstract number sentences. 3 + 3 = 6 Odd + Odd = Even
Doubling of all numbers up to 10	Use practical activities to show how to double a number. 	Draw pictures to show how to double a number 	Begin to relate doubling to x 2 and writing abstract number sentences. 4 x 2 = 8
Repeated grouping / repeated addition	There are 3 equal groups with 4 in each group. Children use Cuisenaire Rods to partition totals into equal trains.  Using Numicon to show 4 x 5: 	Make a necklace with red and yellow beads using three red beads for every yellow bead. Use the bricks to make a tower three times as high as this one:  Children represent the practical resources in a picture and use a bar model. Represent on the number line.	Children start to show recognisable abstract number sentences.  Children are taught about the multiplication 'x' symbol. 3 x 4 = 12 is the same as 4 + 4 + 4 = 12
Children should be here by the end of Year 1			

Year 2

Objective and strategies	Concrete	Pictorial	Abstract
Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24 	Drawing arrays or groups: 3 X ? = 15  Use number lines to show 'repeated addition' and explore commutativity with arrays and number lines to show ending at the same total whichever way calculated. **6 X 5 = 30 and 5 x 6 = 30** **5 + 5 + 5 + 5 + 5 + 5** 	Introducing the inverse operation. Trios can be used to model the 4 related multiplication and division facts. 3 x 5 = 15 5 x 3 = 15 15 ÷ 3 = 5 15 ÷ 5 = 3  Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
TO x O (No regrouping)	Use different resources to create the arrays. 12 x 2 = 24 	Starting to organise and therefore draw arrays in columns and show in a bar model. 24 2 2 2 2 2 2 2 2 2 2 2 2 24 12 12	

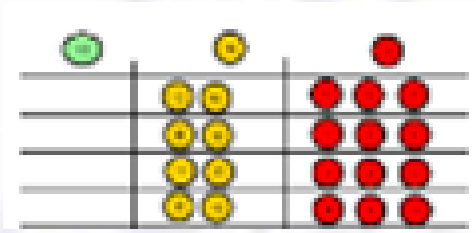
Children should be here by the end of Year 2

Year 3

Objective and strategies	Concrete	Pictorial	Abstract
Multiplying factor (scaling)	Increasing a number of objects by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles. This can be written as: 1 + 1 + 1 = 3 □ scaled up by 5 □5 + 5 + 5 = 15 	Children draw the word problem to find the solution For example, find a ribbon that is 4 times as long as the blue ribbon.  We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 x 0.5 = 1.5.	Children show recognisable number sentences to accompany problem solving. 5 x 4 = 20
TO x O (No regrouping)	Introduction of the **grid method:** 1) Show the link with arrays e.g. 13 x 4  4 rows of 10 4 rows of 3 2) Using Dienes or place value counters in a grid.  	Children can represent the work they have done in a way that they understand.  Show in a bar model. 52 4 4 4 4 4 4 4 4 4 4 4 4 4	Introduction of the **grid method:** Start with multiplying 2 digit by one digit numbers and showing the clear addition alongside the grid, e.g. 5 x14 **X** | **10** | **4** **5** | **50** | **20** **Total** | **50+20=70**

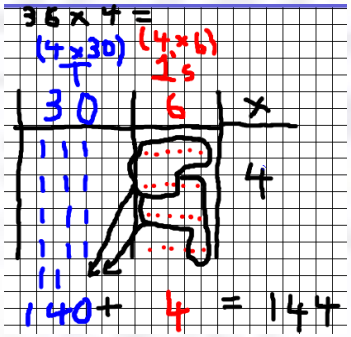
<table>
<tr><th>Objective and strategies</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>TO x O
(with regrouping of ones into tens)</td>
<td>4 x 23 using place value counters (regrouping).

Step 1: Make 4 lots of 23 under place value headings.

Step 2: I have 12 counters in the 1's column. Regroup 10 of these into the Ten's column.

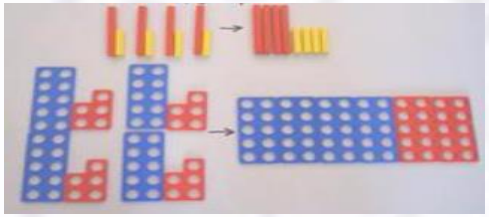
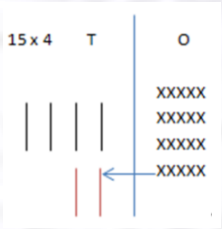
Step 3: Count the number in each column.</td>
<td>Children can represent the work they have done in a way that they understand.

 
</td>
<td>Grid method:

Discuss and explore connections e.g. how multiplying 4 x 3 gives you 12 and the answer is ten times bigger when 4x30.

Regrouping could be shown

<table>
<tr><th>x</th><th>30</th><th>6</th></tr>
<tr><td>4</td><td>120</td><td>24</td></tr>
<tr><td colspan="3">140 +4 = 144</td></tr>
</table>
</td>
</tr>
<tr><td colspan="4">Children should be here by the end of Y3</td></tr>
</table>

Year 4			
Objective and strategies	**Concrete**	**Pictorial**	**Abstract**
solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit,	The Distributive Law says that **multiplying a number by a group of numbers added together** is the same as doing each multiplication separately. This can be shown with Dienes or Numicon. 15X 4 = (10X4=40) +(5x4)=20 Numicon 	Children represent the concrete manipulative in a picture 	Set in context of problems e.g. *If there are 15 passengers in each carriage of a train. If there are 4 carriages, how many passengers in total.* **15 X 4 = (10 x 4) +(5x 4)** **40+20=60** .
TOxTO	**Continue Grid Method to solve problems** *There are 26 strawberries in each of 35 punnets, how many strawberries are there?* × | 30 | 5 20 | 600 | 100 6 | 180 | 30 600 + 100 = 700 180 + 30 = 210 700 + 210 = 910		

TO x O (no regrouping)

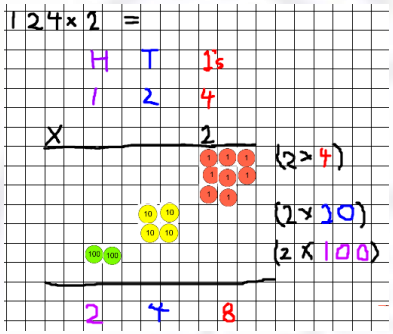
TO x O (with regrouping of ones into tens)

multiply 2- digit and 3-digit numbers by a 1 digit number

Introduction of **compact short** multiplication:

Children can continue to be supported by Dienes or place value counters

Using place value counters to create the sum.

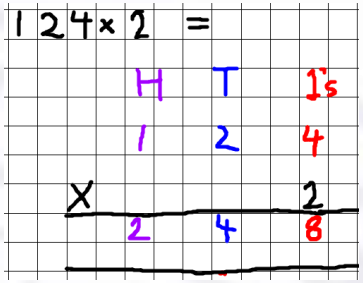


It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

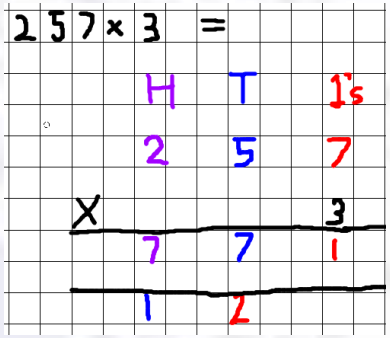
OR: Children could show solutions/ ideas for solving problems using a bar model.

248	
124	124

Without regrouping:



With regrouping:



Set calculations in context of problems

Without regrouping:

If there are 124 sheep in 2 fields, how many sheep in total?

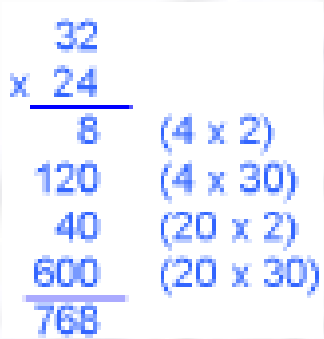
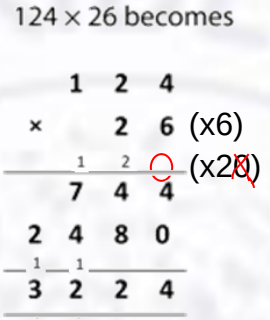
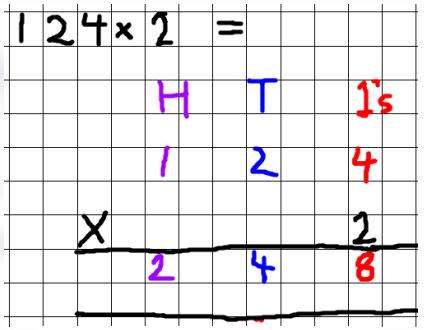
1 2 4 x
2
2 4 8

With regrouping:

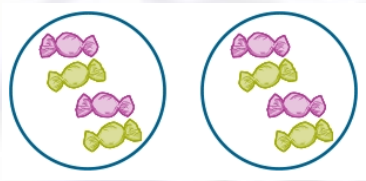
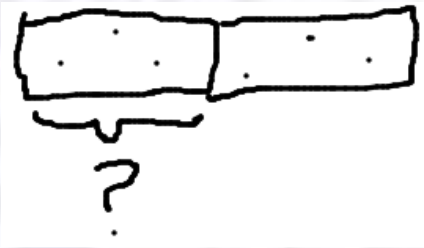
If 3 flights per day carry 257 passengers to Glasgow airport, how many passengers are there in total?

2 5 7 x
3
1 2
7 7 1

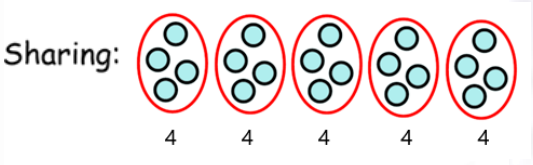
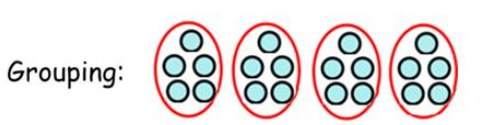
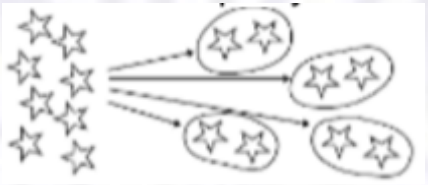
Children should be here by the end of Y4

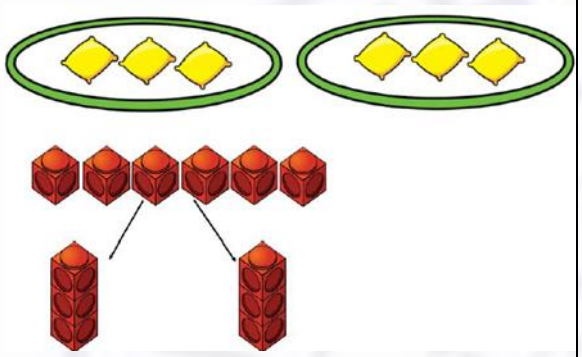
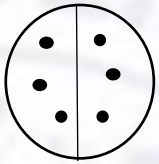
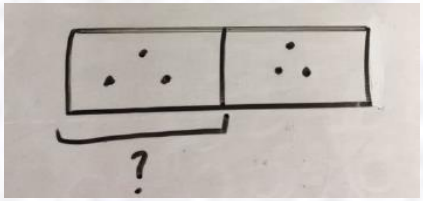
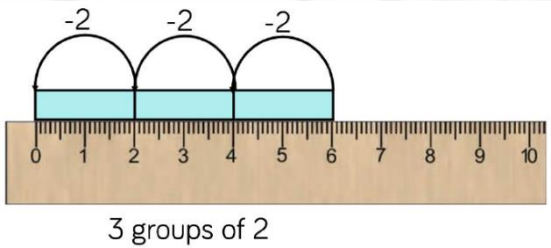
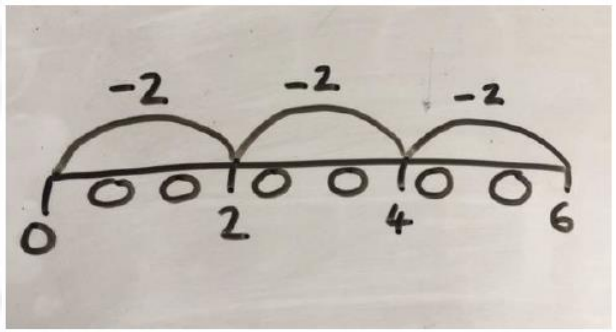
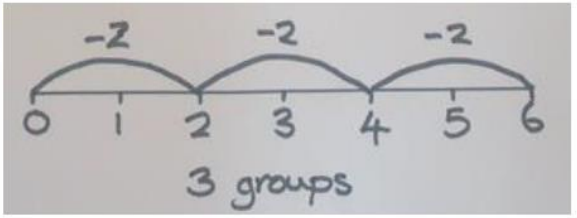
Years 5-6			
multiply numbers up to 4 digits by a one- or two-digit number	**Y5** - Introduction of **expanded long multiplication** with up to 4 x 2 digits. Make it, Draw it, Write it.  32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768	**Y5** - Introduction of **compact long multiplication** with up to 4 x 2 digits**.** Make it, Draw it, Write it.  124 × 26 becomes 1 2 4 × 2 6 (x6) 1 2 (x20) 7 4 4 2 4 8 0 1 1 3 2 2 4	**Y6** – Consolidation of **compact short multiplication** and **compact long multiplication** methods with up to 4 digits by a 2 digit. Including multiplication of decimals. Make it, Draw it, Write it.  124 × 26 becomes 1 2 4 × 2 6 1 2 7 4 4 2 4 8 0 1 1 3 2 2 4

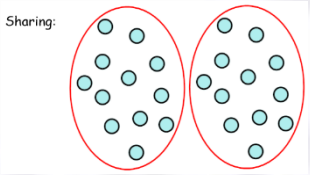
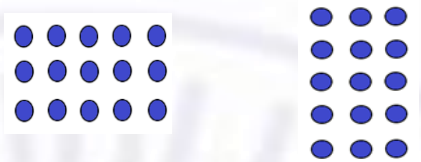
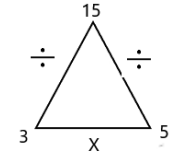
Division

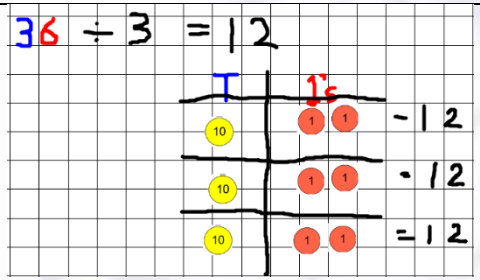
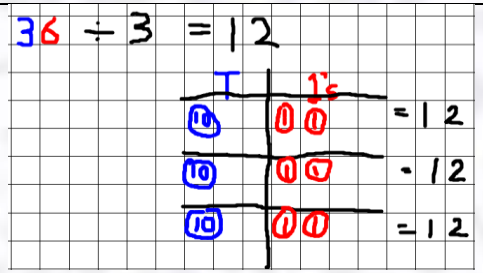
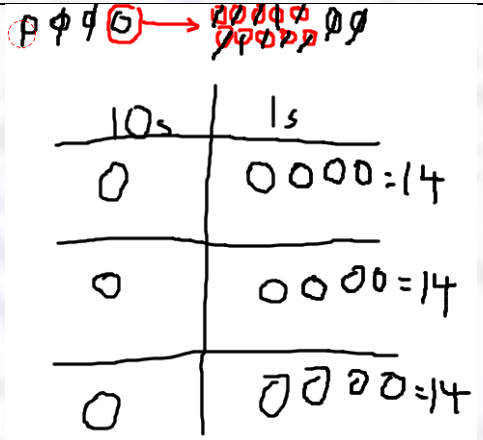
Objectives and strategies	Concrete	Pictorial	Abstract
Equal groups	Children will use apparatus or real objects to group different into equal sized piles. 	Children will draw equal sized groups of objects. 	
Sharing objects into equal sized groups	I have 8 cubes; can you share them equally into 2 groups? 	Represent the idea pictorially and using a bar. 	Introduction of the division sign to mean 'share equally' 6 ÷ 2 = 3
Children should be here by the end of Y1			

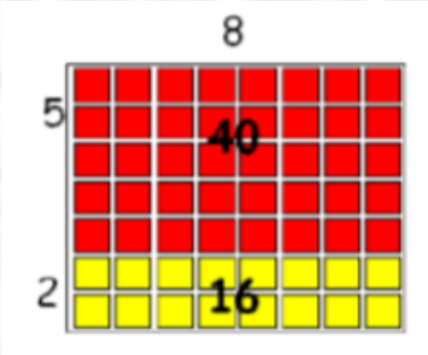
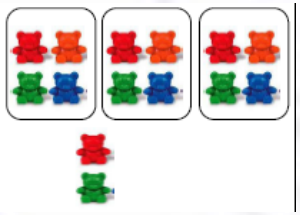
Year 2

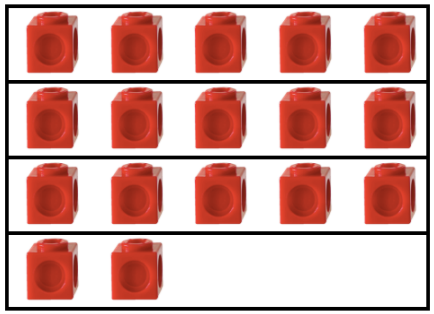
Solve problems which involved sharing or grouping **Know all halves to 10 through grouping and sharing**	**Division by equal share:** Introduce practical problems which the children can physically solve. 1) Look at the number that we are dividing e.g. 20 2) Share this number out equally into section of the number that we are dividing by e.g. 4 3) Count how many there are in each section 20 sweets get shared between 5 people. How many sweets do they each get? 20 ÷ 5 = 4  Can also be connected to showing ¼ of a group or shape. **Division by grouping:** 1) Look at the number that we are dividing e.g. 20 2) Count or draw this many objects 3) How many groups of the number we are dividing by (e.g. take 4 objects and make one group, keep taking 4 to make further groups. 4) Count how many groups you have made 20 ÷ 5 = 4 	Draw a picture to show what happened. **Sharing:**  **Grouping:** 	Link to bar modelling 20 5 5 5 5

Objectives and strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	Share objects into groups. 	Use pictures or shapes to share quantities. Can use pizza method or bar model.  	6 ÷ 2 = 3 Children encouraged to use their 2 times table facts. 
Grouping using repeated subtraction	Repeated subtraction using Cuisinaire rods over a ruler. 	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 24 ÷ ? = 2 	Drawing arrays or groups: 15 ÷ ? = 5  Use number lines to show 'repeated subtraction' and explore commutativity with arrays and number lines to show ending at the same total whichever way calculated. **30 ÷ 5 = 6 and 30 ÷ 6 = 5** **5 - 5 - 5 - 5 - 5 - 5** 	Introducing the inverse operation. Trios can be used to model the 4 related multiplication and division facts. 3 x 5 = 15 5 x 3 = 15 15 ÷ 3 = 5 15 ÷ 5 = 3  Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
Children should be here by the end of Y2			

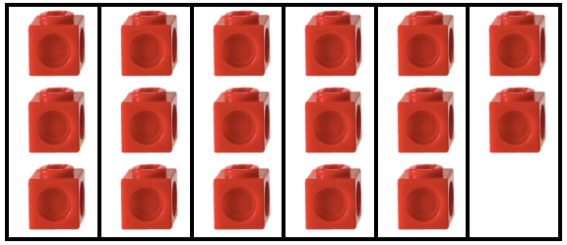
Year 3			
Objectives and strategies	**Concrete**	**Pictorial**	**Abstract**
Sharing **TO ÷ O (with no regrouping and no remainder)**			36 ÷ 3 = 12
Sharing **TO ÷ O (with regrouping and no remainder)**	42 ÷ 3 = 14		Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Objectives and strategies	Concrete	Pictorial	Abstract
Using the distributive law	Create arrays using cubes. Model grouping the sums into different colours or sections. E.g. 56 ÷ 8 can be done as 40 ÷ 8 and 16 ÷ 8	Ch draw the pictorial representation of the array of cubes. 	Write their sum using abstract numbers. 56 ÷ 8 = 7
Sharing **TO ÷ O (no regrouping and a remainder)**	Use concrete objects to share between groups to see how many is left over. 14 ÷ 3 = 4 r 2  Sharing: 17 ÷ 3 = 5 r 2 Divide objects between groups and see how much is left over.	Use a number line to see how many more you need to jump to find a remainder. 13 ÷ 4 = 3 r 1  Draw dots in circles to divide an amount and clearly show a remainder. Sharing: 17 ÷ 3 = 5 r 2  Grouping: 17 ÷ 3 = 5 r 2  Bar model can be used to display whole being divided into equal parts with a remainder left over. 17 ÷ 3 = 5 r 2	Complete written divisions and show the remainder using 'R' for remainder



There are 5 in each group with 2 remaining.

Grouping 17 ÷ 3 = 5 r 2
Put objects into groups of 3 and see how many are left over.

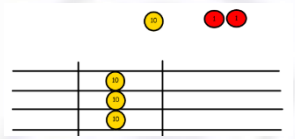
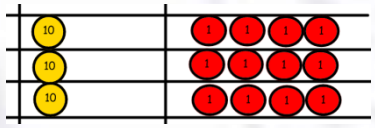


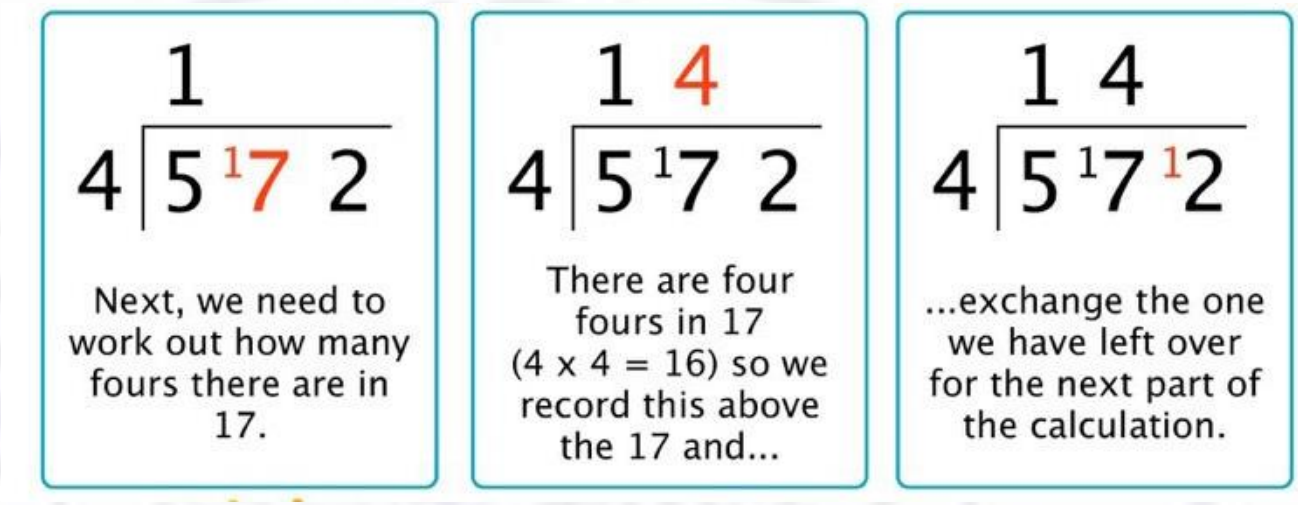
There are 5 groups and with 2 remaining.

?	?	?

5	5	5	R2
17			

Children should be here by the end of Y3

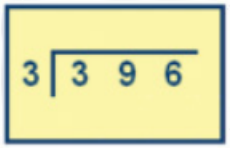
Year 4			
Objectives and strategies	**Concrete**	**Pictorial**	**Abstract**
Division using repeated subtraction/	Start with the biggest place value, sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We **exchange** this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	200 ÷ 6 **1x6 1x6 1x6 10x6=60 10x6=60 10x6=60** 0 2 8 14 20 80 140 200 Answer 33 remainder 2 As you record the division, ask: 'How many sixes in 200?' as well as 'What is 200 divided by 6?' Leading to 200 ÷ 6	96 - 60 10X6 36 30 5 X 6 6 6 1X6 0 16 Answer 16 **Estimation** More than 10X6=60 but less than 20X6=120 This method is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. Children need to recognise that chunking is inefficient if too many subtractions to be carried out. Encourage reducing the number of steps as illustrated in the pictorial stage, when using a number line

Short division method for dividing by numbers within x1 – x12 times tables	
Children should be here by the end of Y4	

Year 5

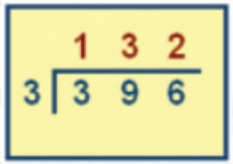
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division

396 ÷ 3 can be written like this:



To work this out, **divide 3 into 396 one digit at a time**, starting from the left with the digit 3 (which represents 300 in the number 396). Put the result of each division on top of the line.

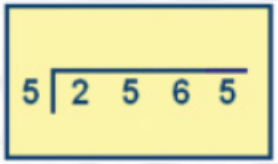
The 3 into 3 goes 1 time exactly,
3 into 9 goes 3 times,
3 into 6 goes 2 times exactly.
396 ÷ 3 = 132



Check if this is correct by multiplying 3 by 132: 3 × 132 = 396

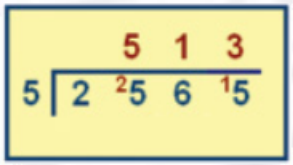
But what if the numbers don't divide exactly? This is where you **carry** numbers.

2,565 ÷ 5 can be written like this:

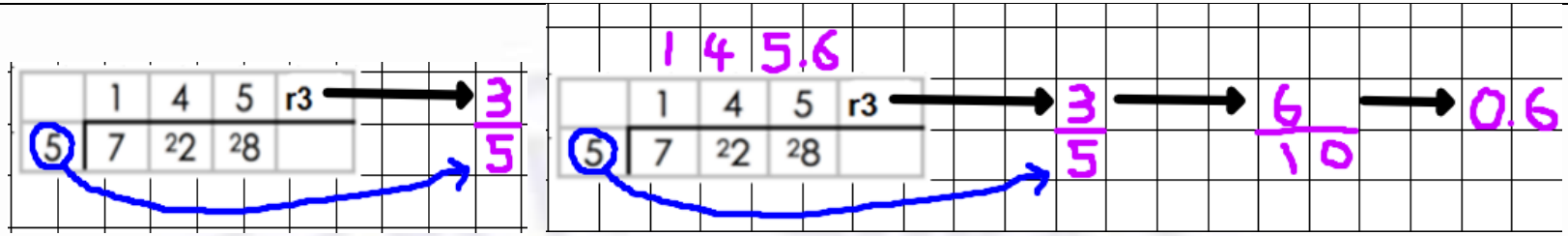


To work this out, **divide 5 into 2,565 one digit at a time**, starting from the left with the digit 2 (which represents 2,000 in 2,565). Put the result of each division on top of the line.

The 5 into 2 won't go so you **carry** the 2 over to the next column, where you now have 25 in the hundreds column,
5 into 25 goes 5 times exactly,
5 into 6 goes once with 1 **remainder** which is carried over to the next column where you now have 15,
5 into 15 goes 3 times exactly.
2,565 ÷ 5 = 513



Check if your answer is correct by multiplying 5 by 513: 5 × 513 = 2,565

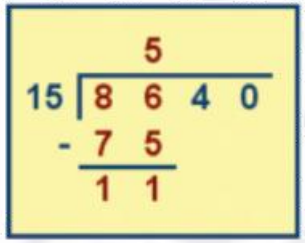
Remainder expressed as a fraction	
Remainder expressed as a simplified fraction	
Remainder expressed as a decimal	
Children should be here by the end of Y5	

Year 6

Long division

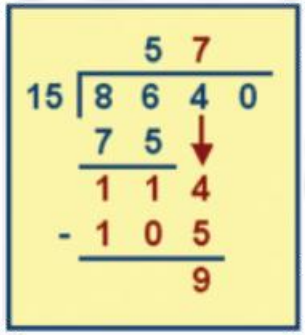
(2 digit divisors)

15 into 8 doesn't go, so look at the next digit.



15 goes into 86 five times, so put a 5 above the 6.
15 x 5 = 75

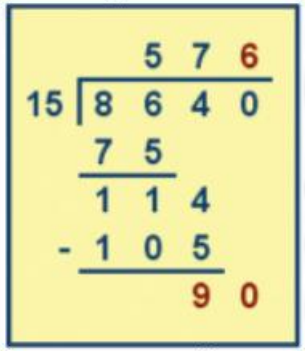
Take that 75 away from the 86 to get your remainder.
86 – 75 = 11



Next, carry the 4 down to make 114.

15 goes into 114 seven times, so put a 7 above the 4.
15 x 7 = 105

Take 105 from the 114 to get your remainder
114 – 105 = 9



Carry the 0 down to make 90

15 goes into 90 exactly 6 times, so put a 6 above the 0

15 x 6 = 90

8,640 ÷ 15 = 576

Children can create a 'skeleton table' next to their calculation to 'hold up' their calculation.

e.g.

30 + 7 = 37 (1)

60 + 14 = 74 (2)

90 + 21 = 111 (3)

120 + 28 = 148 (4)

1 x 37 = 37

2 x 37 = 74

3 x 37 = … etc

37

74

111 … etc